

WATKINS-JOHNSON COMPANY
 3333 HILLVIEW AVENUE
 PALO ALTO, CA 94304
 (415) 493-4141

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DF System Calibration and Correction Techniques

Freq. Avg.	190	200	210	220	230	240	250	260	270	280	290	300	310	320	330	340	350	RMS
40,000	-5	-5																3.283
45,000	-4	-4																2.818
50,500	-4	-3																2.584
60,500	-3	-2																2.368
65,500	-3	3																2.077
70,500	-6	-5																4.102
80,500	-3	-2																1.416

wj WATKINS-JOHNSON
**Manufacturing
 and Office Locations**

United States

SALES OFFICES

CALIFORNIA
 Watkins-Johnson
 2525 North First Street
 San Jose, 95131
 Telephone: (408) 262-1411

Watkins-Johnson
 440 Kings Village Road
 Scotts Valley, 95066
 Telephone: (408) 438-2100

Watkins-Johnson
 831 South Douglas Street
 Suite 131
 El Segundo, 90245
 Telephone: (213) 640-1980

International

MARYLAND
 Watkins-Johnson
 700 Quince Orchard Road
 Gaithersburg, 20878
 Telephone: (301) 948-7550

MASSACHUSETTS
 Watkins-Johnson
 5 Militia Drive
 Suite 11
 Lexington, 02173
 Telephone: (617) 861-1580

OHIO
 Watkins-Johnson
 2500 National Road
 Suite 200
 Fairborn, 45324
 Telephone: (513) 426-8303

DISTRICT OF COLUMBIA
 Watkins-Johnson
 700 Quince Orchard Road
 Gaithersburg, MD 20878
 Telephone: (301) 948-7550

GEORGIA
 Watkins-Johnson
 4250 Pennter Park, South
 Suite 123
 Atlanta, 30341
 Telephone: (404) 458-9907

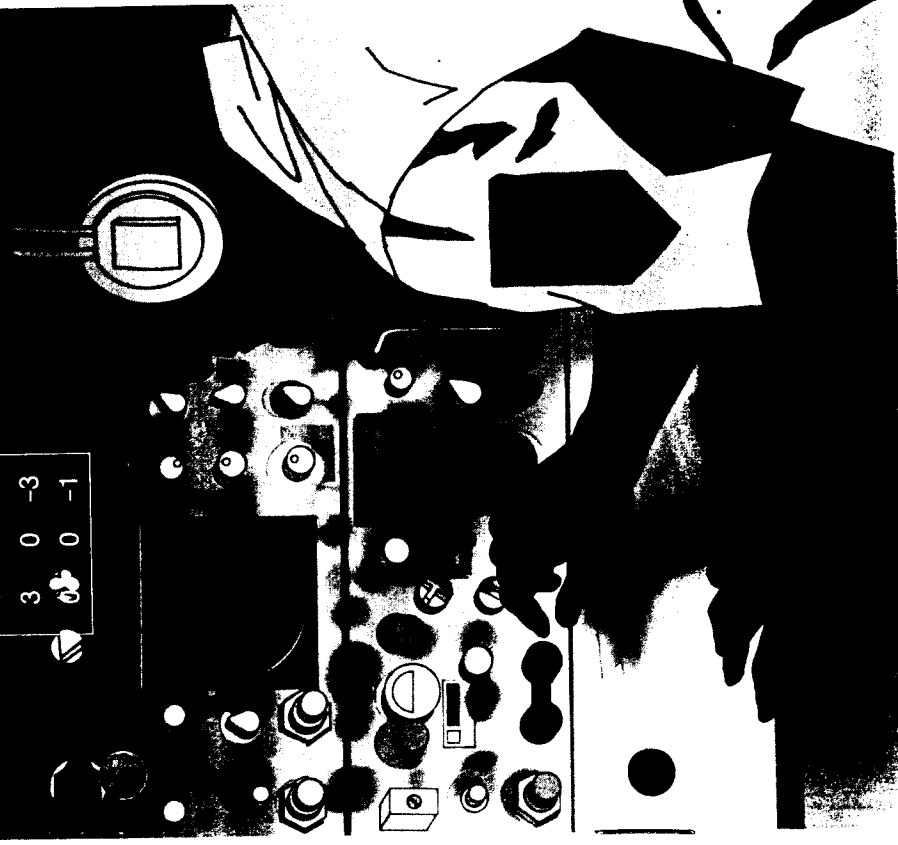
ITALY
 Watkins-Johnson
 S.p.A.
 Piazza G. Marconi, 25
 00144 Roma-EUR
 Telephone: 59 45 54
 Telex: 612276
 Cable: WJ ROM I

UNITED KINGDOM
 Watkins-Johnson
 Dedworth Road
 Oakley Green
 Windsor, Berkshire SL4 4LH
 Telephone: (07535) 69241
 Telex: 847578
 Cable: WJUKW-WINDSOR

**GERMANY, FEDERAL
 REPUBLIC OF**
 Watkins-Johnson
 Manzingerweg 7
 8000 Muenchen 60
 Telephone: (089) 836011
 Telex: 529401
 Cable: WJDBM-MUENCHEN

TEXAS
 Watkins-Johnson
 9215 Markville Drive
 Dallas, 75243
 Telephone: (214) 234-5396

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The primary purpose of this article is to describe how to construct a reliable correction table for radio direction-finding applications. The information presented will also enable one to test a direction finding (DF) unit to the manufacturer's specifications, as well as test an active DF site for degraded performance due to aging or extreme environmental conditions.

The correction tables described here are used to increase the overall accuracy of the DF system. This aspect becomes important when a DF cut is being made on a transmitter that is 30 or 40 miles from the DF site, or during triangulation operations.

A large part of this discussion is dedicated to statistics and data reduction techniques. These are the tools needed to perform accurate direction finding and to calculate the amount of accuracy. For a more detailed description of the techniques described, a standard college-level statistics text and a text of mathematical methods that includes a section on curve fitting are recommended.

The first section is on data collection. Procedures for constructing of a test area and for collecting data are presented. The key idea is to keep track of all the information.

The second section describes how to test the data that has been collected. The data can be compared with past performance to determine if the DF is in need of recalibration. A new DF system can be tested to see if it is as good as an older system was or if it meets manufacturer's specifications. In the previous case, most of the tests will be performed against specific frequencies. In the second case, where the DF system is being tested for overall accuracy, many sets of frequencies are combined. The first step in doing any data testing is to calculate the standard

deviation (SD). The Chi-squared test (χ^2) can then be used to test the SD from the data to the SD of the historical performance or a specification.

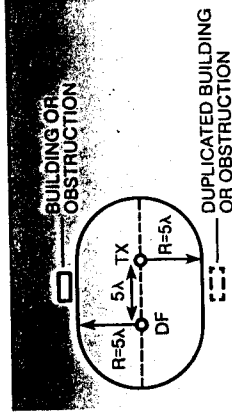
After the data has been collected and tested, the third section shows how to break the data of each frequency down to the desired azimuth increment. This section is useful if graphical representation is desired as a form of correction table. Several algorithms are presented for computer implementation of these procedures.

The concluding remarks of this article can be used as a check list for constructing correction tables or for testing DF systems.

Data Collection for System Calibration

The location used to test a DF system depends largely on the frequency range and the type of information desired. If the DF system will be at a fixed location (usually called a fixed-site DF) and if the data will be used to generate a correction table, then the actual fixed-site location is the best test location. If the unit is man-portable or van-mounted, and if the basic system errors are being measured to create a correction table, then an anechoic chamber is more desirable.

If an anechoic chamber is not available, a large open area is the next most suitable site. The area should be at least 15 wavelengths (of the lowest frequency to be tested) long and 10 wavelengths wide, to reduce the signal strength of any reflected wave. All reflecting surfaces around the DF antenna should be duplicated on the opposite side of the area about the line joining the transmitting antenna to the DF antenna. See Figure 1 as an example. If it is impractical to duplicate the reflection, then it



TX = TRANSMITTER
DF = DIRECTION FINDER

Figure 1. Small field with obstructions duplicated.

may be preferable to change the dimensions to those indicated in Figure 2.

The DF antenna and the transmitting antenna need to be at least five wavelengths apart to insure that the DF antenna is in the "far field." If the DF antenna is closer, the curvature of the wavefront can induce extra errors. To help reduce the errors caused by reradiations of surfaces near the transmitting antenna, the overall length of the test area should be increased, if possible.

It is easy to diagnose reflections or reradiations during a DF test. If there are frequencies where all the readings are characterized by a positive or negative average error, yet the errors, when plotted, form a smooth curve, there is most likely a reflection or a reradiation present. To test this hypothesis, run the same test a second time with the DF antenna in a new location (move the DF antenna a full wavelength in any direction). If the average error changes by more than a degree or two, then the error was caused by reflections or by reradiations.

The previous paragraph assumes the DF unit is one of the many phase-measurement based systems (this would include doppler and pseudo-doppler techniques). If an amplitude

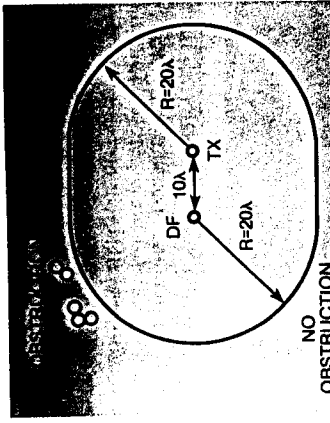


Figure 2. Large field with obstructions in safety region.

measurement DF unit is used, the reflection can often be received as a second bearing. If a directional antenna is being swept 360° and a CRT is used to display the signal strength, the reflection can easily be identified as the signal not coming from the transmitting antenna. With loop antennas, because the signal is coming from two directions, there will never be a complete null. There may be two null points, the lowest of which should be the transmitter. The other null will be the reflection.

An important goal during the DF test is to keep all conditions as constant as possible and to record all pertinent data. If the tests last more than one day, and each night the gear must be stored, be sure the area is marked so the antennas can be replaced exactly where they were. Make a sketch of the area and any obstructions, to scale if possible, that includes any trees, buildings, sign posts, trucks, etc. When the area is being set up at a later date, check the sketch to see if anything has changed and make a note of it. When the data is being reviewed, the sketch may help identify reflections.

When tabulating data, it is important to be consistent in the method used to determine the difference between the

true LOB and DF reading. The procedure to use is:

$$\text{CORRECTION} = (\text{true LOB}) - (\text{DF reading})$$

Using this method, the values read from the correction table can be added

to the "DF reading" to determine the true LOB.

Figure 3 is an example of a useful data sheet. It allows for all important data to be recorded, including a couple of lines for comments.

FREQ	SIGNAL STRENGTH	DB	SN	SN	DATE	TIME	COMMENTS
360							
350							
340							
330							
320							
310							
300							
290							
280							
270							
260							
250							
240							
230							
220							
210							
200							
190							
180							
170							
160							
150							
140							
130							
120							
110							
100							
90							
80							
70							
60							
50							
40							
30							
20							
10							
0							
ANG							
LOG #							

Figure 3. Sample data collection log.

Data Testing

Once the data is collected, it should be tested to determine if it is acceptable data. Acceptable data is data that matches any existing data, from the system, within a given tolerance or quality level. The general measures of accuracy are the RMS and the Standard Deviation calculations. Both of these calculations will be described below. The measure of tolerance is performed with a Chi-squared calculation. This will also be discussed below. For brevity, the material presented below will be functionally oriented. For a more rigorous discussion of this material, refer to the reference listed on the last page.

The Standard Deviation

Once the data has been collected and tabulated, it must be determined if the data is within the manufacturer's specifications or if the data is usable for correction tables.

The most common test of overall DF accuracy is represented by the RMS correction criterion. RMS correction is an abbreviation for Root Mean Square. The value of RMS is the measure of the average deviation of the corrections. The equation used to calculate RMS is:

$$\text{RMS} = \sqrt{\frac{\sum_{i=1}^n x_i^2}{n}}$$

where x_i = (true LOB) - (DF reading) and n is the total number of data points.

The RMS calculation is a good measure of DF accuracy, but it does not take into account any verified reflections that may be present. As discussed earlier, it is almost impossible to remove all reflections, and the effects of any such reflection is to shift all the data points by some average value. The average offset

(\bar{x}) can be calculated by averaging all the correction values for a particular frequency. To compensate for the offset caused by the reflection, subtract the average offset from x_i before squaring and adding it to all the other values. This would make the RMS equation look like:

$$\text{RMS (compensated)} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$$

This equation is very similar to the equation for standard deviation (SD) in statistics, and can be viewed as the RMS value about the average. (Some statistics texts refer to this equation as the RMS deviation.) In statistics, this would be called a "biased" SD. To obtain a more reliable measure of accuracy and to be able to use other statistical tests, one more modification should be made in the equation,

$$\text{SD} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

This is the "unbiased" form of the equation. (Whenever Watkins-Johnson specifies an RMS value for its VHF/UHF DF units, it can be treated as the standard deviation.)

The following is a demonstration of the process of calculating the RMS and SD values for a particular frequency. In Table 1, the first column is the measured LOB (from the zero reference on the DF antenna) to the transmitting antenna. The second column represents the bearing the DF processor gave for each true LOB. Notice that at a true LOB of 360 degrees, the DF reading was 0; the Watkins-Johnson VHF/UHF DF units will respond up to 359 degrees, a bearing of 360 is displayed as 0. The third column is the correction column. When the correction column is added to the

DF reading column the resulting value will be the true LOB. As shown earlier, the correction is calculated by subtracting the DF reading from the true LOB. In any of the following examples, only the true LOB and correction columns will be presented.

Using the correction column as the data values, the RMS value is calculated as:

$$RMS = \sqrt{\frac{(0)^2 + (-1)^2 + (-1)^2 + \dots + (3)^2 + (3)^2 + (2)^2}{36}} = 1.58^\circ$$

The average offset is:

$$\bar{x} = \frac{(0) + (-1) + (-1) + \dots + (3) + (3) + (2)}{36} = -0.06^\circ$$

The SD value can then be calculated as:

$$SD = \sqrt{\frac{(0 + 0.06)^2 + (-1 + 0.06)^2 + (-1 + 0.06)^2 + \dots + (3 + 0.06)^2 + (2 + 0.06)^2}{36 - 1}} = 1.60^\circ$$

Notice that the SD is larger than the RMS value. This will always be true when the average offset is approximately zero (as is true in this case).

The following is an example of the behavior of the RMS and SD values in the presence of a reflection. First, refer to Table 2, column A for the data values without a reflection.

$$RMS = \sqrt{\frac{(4)^2 + (3)^2 + (1)^2 + \dots + (0)^2 + (2)^2 + (3)^2}{36}} = 2.92^\circ$$

Taking the average of the data gives:

$$\text{Average Offset} = \bar{x} = \frac{(4) + (3) + (1) + \dots + (0) + (2) + (3)}{36} = -0.17^\circ$$

The number of data values is n = 36. Notice the data value at 360° is not included. To include it would be the equivalent of counting 0° twice.

Using \bar{x} in the SD equation we have:

$$SD = \sqrt{\frac{(4 + 0.17)^2 + (3 + 0.17)^2 + \dots + (2 + 0.17)^2 + (3 + 0.17)^2}{35}} = 2.95^\circ$$

which is a little larger than the RMS value, but the values are very close.

To see the effects of a reflective surface in the test area, a reflector was added to the original equipment setup. The resulting data is shown in Table 2 column B, and a plot of the data is shown in Figure 4. It is important to remember that this is the same physical set up as used in column A except with the reflecting surface.

From Figure 4 it can be seen that the average offset of the corrections is some negative number. Taking the average of the actual data values from Table 2 we have:

$$\bar{x} = \frac{(-8) + (-10) + (-12) + \dots + (-12) + (-9) + (-8)}{36} = -12.75^\circ$$

The RMS value will be:

$$RMS = \sqrt{\frac{(-8)^2 + (-10)^2 + (-12)^2 + \dots + (-12)^2 + (-9)^2 + (-8)^2}{36}} = 13.14^\circ$$

which is totally different than the RMS from column A.

REFL	TRUE LOB	DF Reading	Correc- tion
	0	0	0
	10	11	-1
	20	21	-1
	30	30	0
	40	40	0
	50	49	1
	60	59	-1
	70	60	0
	80	80	0
	90	91	-1
	100	103	-3
	110	113	-3
	120	123	-3
	130	132	-2
	140	141	-1
	150	150	0
	160	159	1
	170	170	0
	180	181	-1
	190	192	-2
	200	202	-2
	210	211	-1
	220	220	0
	230	228	2
	240	238	-2
	250	249	-1
	260	259	-1
	270	270	0
	280	281	-1
	290	291	-1
	300	300	0
	310	309	1
	320	317	-3
	330	327	-3
	340	337	-3
	350	348	-2
	360	360	0

Table 1. Sample data set.

ANGLE	DF	Correc- tion
0	0	0
10	11	-1
20	21	-1
30	30	0
40	40	0
50	49	1
60	59	-1
70	60	0
80	80	0
90	91	-1
100	103	-3
110	113	-3
120	123	-3
130	132	-2
140	141	-1
150	150	0
160	159	1
170	170	0
180	181	-1
190	192	-2
200	202	-2
210	211	-1
220	220	0
230	228	2
240	238	-2
250	249	-1
260	259	-1
270	270	0
280	281	-1
290	291	-1
300	300	0
310	309	1
320	317	-3
330	327	-3
340	337	-3
350	348	-2
360	360	0

Table 2. Sample data set.

Taking into account the average offset and calculating the SD:

$$SD = \sqrt{\frac{(-8 + 12.75)^2 + (-10 + 12.75)^2 + \dots + (-9 + 12.75)^2 + (-8 + 12.75)^2}{35}} = 3.24^\circ$$

which is approximately equal to the SD of column A.

The SD of corrections 360° around the antenna at a given frequency is a good measure of the accuracy at that particular frequency. This value of SD can then be used as the "error range" in triangulation, as described in the Watkins-Johnson *Tech-notes on Improving System and Environmental DF Accuracy*. We can also use this information to create the initial correction table. This data will then become the "historical data" to which future data can be compared. It is always a

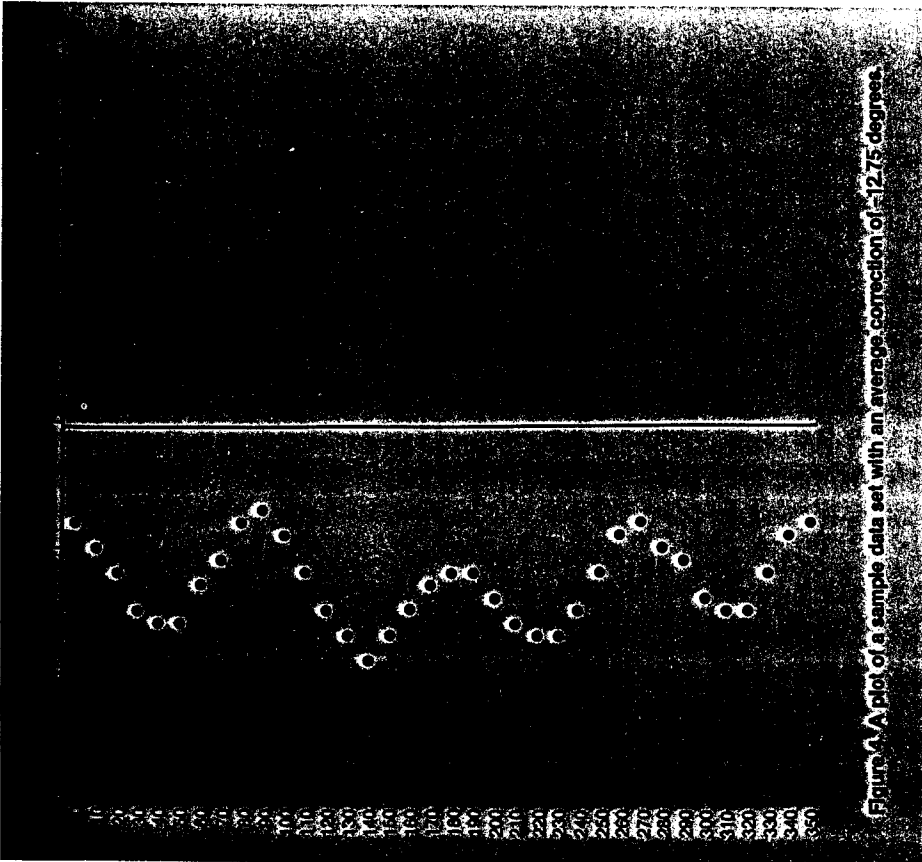


Figure 1 A plot of a sample data set with an average correction of 12.75 degrees.

good idea to update the correction table periodically, combining the new data with the original data.

When testing a manufacturer's specification, the SD should be calculated from a very large number of data points. A good rule of thumb is to pick 5 or more frequencies of interest, determine the correction at ten-degree increments, and use all these values to determine the system SD.

Now the question arises: is an SD of 3.24° an acceptable indication of the previous SD of 2.95°? There are statistical tests to determine the acceptability of the 3.24°.

The Chi-squared Test

There are several statistical tests to compare SD of a data set either against a specification, or a SD of another data set. One of these tests is the Chi-square (χ^2) test. The Chi-square test can be viewed as a tolerance test of the SD based on the confidence level.

When a decision is made to accept or reject a data set, two possible types of errors can occur. The first is to reject a data set that is satisfactory; the second is to accept a data set that is unsatisfactory. Statisticians call these errors Type 1 and Type 2 errors, respectively. If the probability of one type of error is

specified, there are ways of determining the probability of the other type of error. The relationship between Type 1 and Type 2 error is such that if the probability of accepting a satisfactory data set is increased, the probability of accepting an unsatisfactory data set is also increased.

An acceptable compromise (and general rule of thumb), which allows the acceptance of most of the satisfactory data sets and the rejection of most of the unsatisfactory data sets, is to set the probability of accepting a satisfactory data set to 95%. This is the value that statisticians call the "confidence value." A confidence level of 95% says that only 5% of the satisfactory data will fail the Chi-square test.

The Type 2 error can be calculated using the desired confidence level and the number of data values taken. The larger the number of data values, the lower the probability of accepting an unsatisfactory data set. Because the calculation for determining the probability of a Type 2 error is somewhat tedious, it will not be included here. Many text books on statistical decision making (such as the first two texts included in the references) will give "Operational Characteristic" curves (or OC curves, as they are sometimes called), where the probability of the Type 2 error can be read directly from a graph. As an example of the sensitivity of the Type 2 error to the number of data values, given a 95% confidence level, a 5° SD will be accepted as 3°, 10% of the time when 18 data values are taken, 2% of the time when 36 data values are taken, and approximately 0% of the time when 72 data values are taken. The rule of thumb for testing a DF system was made to reduce the possibility of accepting a DF unit that is not performing to the specification (The rule of thumb uses 5 separate frequencies and 10° azimuth intervals).

The two important pieces of information needed to perform the χ^2 test is the number of data points taken and the calculated standard deviation. The equation to calculate the χ^2 is:

$$\chi^2 = \frac{(n-1) s^2}{\sigma^2}$$

where,
 n is the number of data points,
 s is the calculated standard deviation of the data, and
 σ is the desired standard deviation.

For an accurate test, n should be larger than 30. A practical value is n = 36. This represents a correction reading every ten degrees.

Once a value for χ^2 has been calculated, the acceptance value must be determined. This value is based on the number of independent readings, and is represented by γ (the greek letter "nu"), which is equal to the number of data values minus one (n-1). Using the 95% confidence level mentioned above, the critical value can then be found in a Chi-squared table under the 0.95 (95 percent) of the area covered (or 0.05 of the tail of the distribution).

Table 3 has several critical values for a confidence factor of 95 percent. The critical value is the value of χ^2_0 , the worst-case value that is acceptable. The following example uses the SD value found in the previous example and determines if it is an acceptable data set.

The SD value calculated was SD = 3.24 degrees, with 36 data points used to calculate it (from 0 degrees to 350 degrees). The value we want to test to is 2.95 degrees SD; so, n = 36, s = 3.24, σ = 2.95, and

$$\chi^2 = \frac{(36-1)(3.24)^2}{(2.95)^2} = 42.22.$$

Since $\gamma = n-1 = 35$ (one reading every 10 degrees), from Table 3, $\chi^2_0 = 49.76$. Since

42.22 is less than 49.76 ($\chi^2 < \chi_0^2$), the value of 3.24° is acceptable as a 2.95° SD specification. (This is because all the data points could not be considered.)

This last example tested the SID of a data set to a historical specification (a value determined from past experience). The same process is used to test a manufacturer's specification, with the exception of the number of data points. To test a manufacturer's specification, it is customary to choose five or more frequencies and determine a correction value every ten degrees. This represents 180 data values; it is not uncommon to use as many as 10,000 data values. Table 3 has the χ_0^2 values for 503, 2,519, and 11,519 data values. Because the graph of χ_0^2 becomes approximately linear in this area, the linear interpolation to a particular number of data values will be adequate to verify the system's performance.

Degrees Between Readings	γ	χ_0^2 for 95% Confidence
20	17	27.59
10	35	49.76
5	71	90.53
3	119	145.10
NA	503	555.84
NA	2,519	2,636.25
NA	11,519	11,769.00

Table 3. χ_0^2 values for the χ^2 test.

RMS and SD are useful in proving the acceptability of the data, but what happens if there are two data sets being compared to one another? Two data sets that have less than a 3-degree RMS error and both pass the χ^2 test may still be critically different in nature. This phenomenon can be seen when direction finding is performed at different frequencies. To get an accurate data acceptance, the data needs to be com-

pared for correlation. There are mathematical tools for determining the amount of correlation, but for most DF work, it is easier and faster to look at a plot of the correction values. To check the correlation of two data sets, first plot the data, then compare the plots for the basic shape, position and number of any lobes (i.e., Figure 4 is a four-lobe correction plot).

The Correction Table

The environment is dynamic, always changing, which can cause small variations in the received LOB. Because of these changes, for both mobile and fixed-site DF's, it is advisable to repeat the DF measurements several times and average the corrections of the acceptable DF data sets together to form a more accurate correction table. Table 4 shows this for data taken on 3 separate dates. All three data sets pass the 3-degree χ^2 test and the curves are similar; the differences could have been caused by the weather or differences in operator technique. By taking the average of the 3 data sets, the miscellaneous errors will begin to cancel out and the real environmental error can be determined. Caution: It is not always as easy as this to determine if the data sets are good, but the technique is valid.

Once a number of data sets have passed the χ^2 test, and the data plots of common frequency have passed a correlation test and have been averaged, a correction table can be produced. A correction table is a collection of data sets presented in a convenient format for referencing the frequency and the DF bearing. Some correction tables include graphs or RMS values to give an idea of the kind of accuracy that can be expected at given frequencies.

A correction table for a fixed-site DF

ANGLE	JUN 71	AUG 71	NOV 72
0	2	2	3
10	2	3	2
20	3	3	3
30	3	3	2
40	3	4	3
50	4	4	3
60	4	3	2
70	3	3	2
80	3	2	2
90	2	2	2
100	1	1	0
110	1	1	0
120	1	0	0
130	0	1	0
140	0	1	0
150	1	0	0
160	1	1	2
170	2	1	2
180	2	2	3
190	3	3	2
200	3	3	2
210	4	4	3
220	4	3	2
230	3	3	2
240	2	3	2
250	2	2	1
260	1	2	1
270	2	2	1
280	2	2	1
290	2	2	0
300	1	1	0
310	1	1	0
320	0	0	1
330	0	0	2
340	1	1	2
350	2	1	2
360	2	2	3
RMS	1.134	1.180	0.964

Table 4. Averaging similar data sets for a more accurate correction table.

system should display the true corrections measured at a given angle and frequency. A mobile DF correction table would be more useful if the corrections are tabulated around the average. As described earlier, an average offset is generally caused by a reflection; that reflection will probably not be constant as the DF is moved from one environment to another.

Figure 5 is a sample of a small correction table from 40 MHz to 80 MHz. This correction table was made for a mobile application. Notice the blank column for the average and the double spacing between lines. Besides readability, this provides room for the DF operator to manually update the table if he is going to be in one location for an extended period of time.

Figure 5 Sample correction table

Data Reduction

Once all the data is collected, the data can be formatted as in Figure 5. There are times when a higher degree of resolution in azimuth position is desirable or a smooth plot of the data would give a better insight to the location of the transmitted signal. In these cases, there are methods to manipulate the data to give the desired resolution.

For example, if the correction table is set up with 10-degree increments as in Figure 5, a DF reading of 93 degrees may be corrected by interpolation. Interpolation is also used to determine the correction value between two frequencies at a given azimuth.

Although there are many ways to interpolate data, the most common is linear interpolation. This type of interpolation takes into account the slope between only two data points. Linear interpolation is also the starting point for the other forms of interpolation discussed in this paper.

For example, given an RF frequency of 70.5 MHz and a DF reading of 93 degrees, what is the probable true LOB (using Figure 5 as the correction table)?

The first step in doing a linear interpolation is to find the slope formed by connecting a straight line to the two correction values.

$$s = \frac{\text{Correction at } 100^\circ - \text{Correction at } 90^\circ}{100^\circ - 90^\circ} = \frac{(-3) - (0)}{10}$$

$$s = -3$$

The equation to use next is:

$$C_k = (s)(k-n) + k$$

where,

C_k is the correction value for the DF reading (C_{93} in our case)
 s is the slope ($s = -3$ as calculated above)

k is the DF reading (93° in our case)
 n is the correction table LOB that is on the zero side of k and is one of the two LOBs that are on either side of k (90° in this case)

$$\text{so, } C_k = C_{93} = (-3)(93-90) + 93 = 92.1^\circ$$

If the DF unit has a 1-degree resolution, then the true LOB is rounded to 92 degrees. The resolution obtained from the linear interpolation equation should be no greater than that of the original DF bearing.

Linear interpolation is monotonic. It serves well for connecting one data value to another, but it does not take trend into account. To take the data trend into account, the slopes of the two lines connecting three data values have to be considered. This process is called curve fitting.

There are many different algorithms that can be used for curve fitting. Most of them use some sort of weighted averaging technique. The different techniques can be viewed as different types of numerical filters with differing numbers of poles. One good technique is the binary weighted average, or 1-2-1 filter. If we extend the notation used for the linear interpolation equation such that C_n is the n th data value, the equation for this type of filter is,

$$C'_n = \frac{\text{The new value of } C_n}{4} = \frac{(1)(C_{n-1}) + (2)(C_n) + (1)(C_{n+1})}{4}$$

This equation can be expanded to cover enough data points to get a smooth curve. A seven-pole filter does very well when trying to curve fit seven interpolated points between two, ten-degree increment, data values. The equation for this is,

$$C'_n = \frac{(1)(C_{n-3}) + (2)(C_{n-2}) + (4)(C_{n-1}) + (8)(C_n) + (4)(C_{n+1}) + (2)(C_{n+2}) + (1)(C_{n+3})}{22}$$

Notice that all the coefficients are symmetrical about the C_n term and they progress as powers of two, that there are seven terms (hence seven poles), and that the denominator is the summation of all the coefficients.

The shortcoming of any form of averaging is that once the filtering process is performed on a data value, the data loses its true value. To regain the exact data value, a correction algorithm is needed.

A simple correction algorithm is to take the difference between two original data values and two curve-fitted data values, and use the slope to correct the intermediate values. To find the slope s ,

$$s = \frac{(E'_m) - (E_n)}{m - n}$$

where $E'_m = C'_m - C_m$ and $E_n = C_n - C'_n$, and where m and n are the LOBs at which the original data was taken, and $m < n$ (i.e., from Figure 5; $m = 90^\circ$, $n = 100^\circ$).

The correction equation can now be written as:

$$C'_k = \text{New value of } C'_k = E_n - (s)(k-n) = C'_k$$

where C'_k is the corrected value of C'_k and k is the position between m and n .

The reason the linear interpolation, numerical filter, and correction techniques are called algorithms is that the equation must be repeated for all C_n (or C'_n). Figure 6 shows a block diagram of how this can be accomplished. It is identical for all three processes.

Now that the mathematical tools have been developed, they may be combined into a working unit. Figure 7 shows the proper sequence for using the procedures, and Figure 8, 9 and 10 show the results of this process. Figure 8 is a point-to-point data value plot. The data was taken in 10-degree increments. Figure 9 is the same data after linear interpolation has been performed. Figure 10 is a plot of the same data after going through the curve fitting and correction algorithms. When looking at these plots, notice that they are a convenient means of presenting the correction for frequency and azimuth. This is the requirement of a correction table. Therefore, these plots can themselves be used as a form of correction table. Likewise, the corresponding tables of values used to generate the graphs can also be used as correction tables.

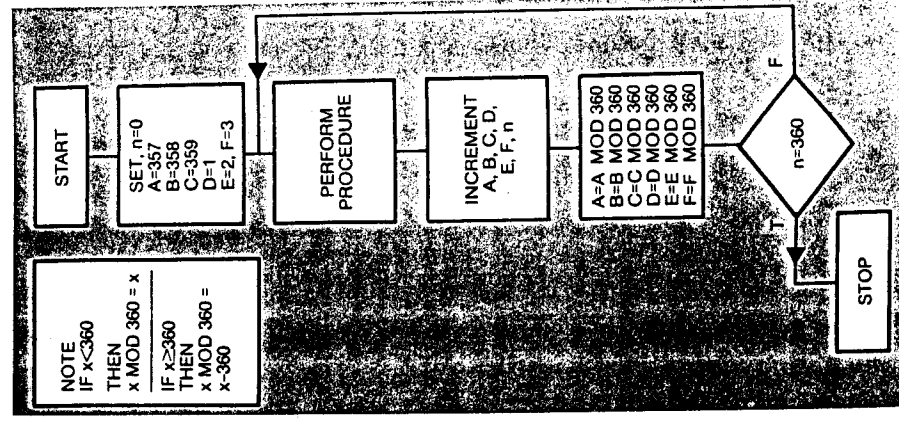


Figure 6. Flow chart for interpolation, curve-fitting, and correction algorithms.

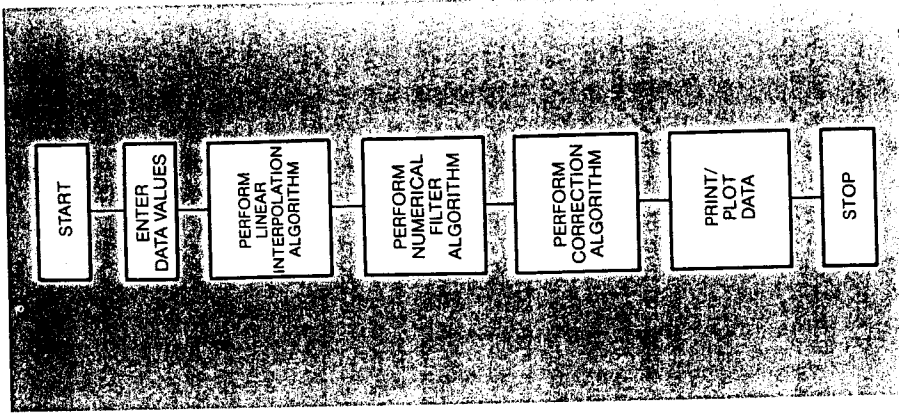


Figure 7. Flow chart for data reduction Procedure #1.

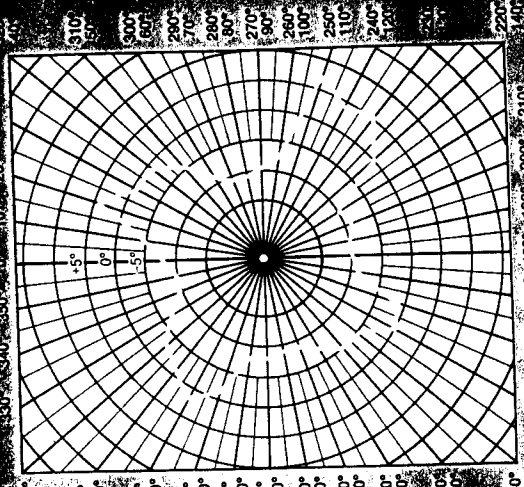


Figure 8. Point-to-point plot.

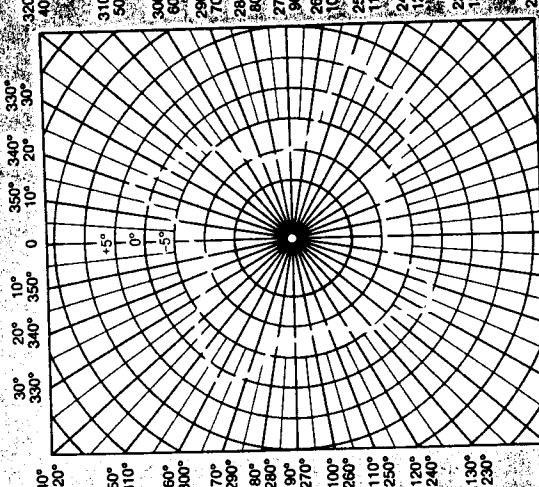


Figure 9. Data plot with linear interpolation.

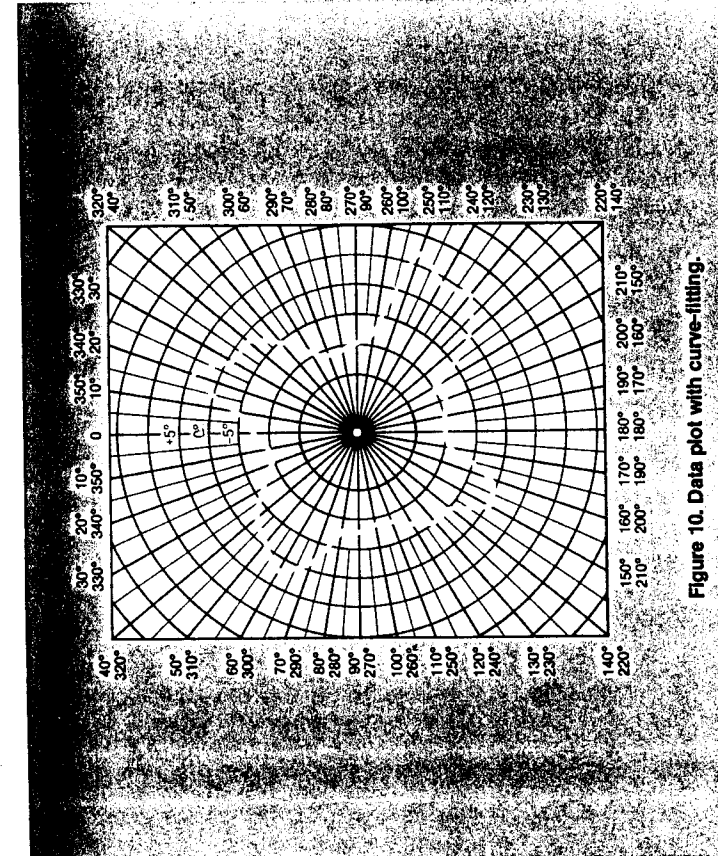


Figure 10. Data plot with curve-fitting

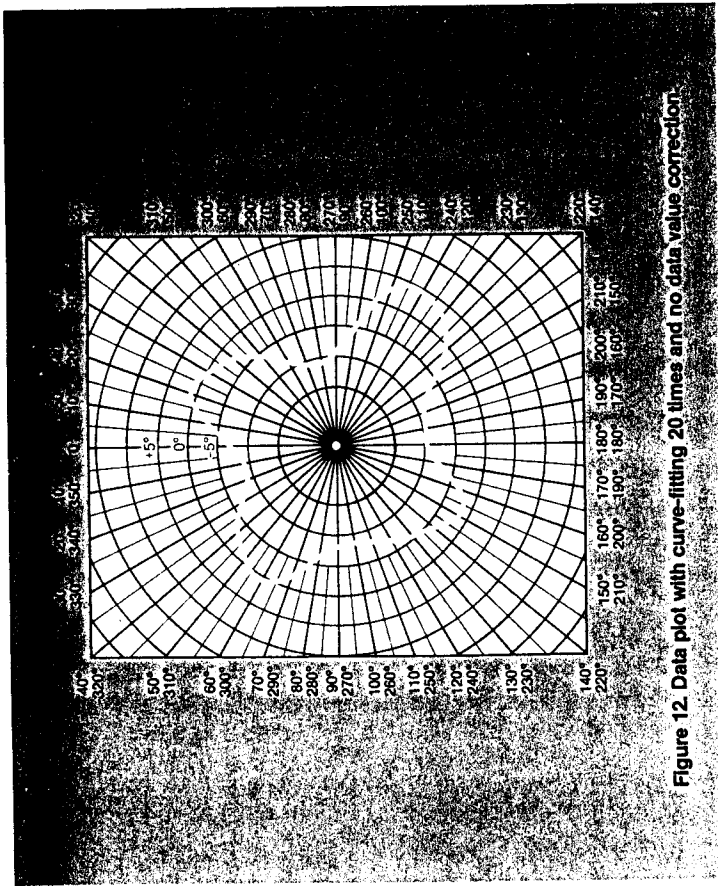


Figure 12. Data plot with curve-fitting 20 times and no data value correction

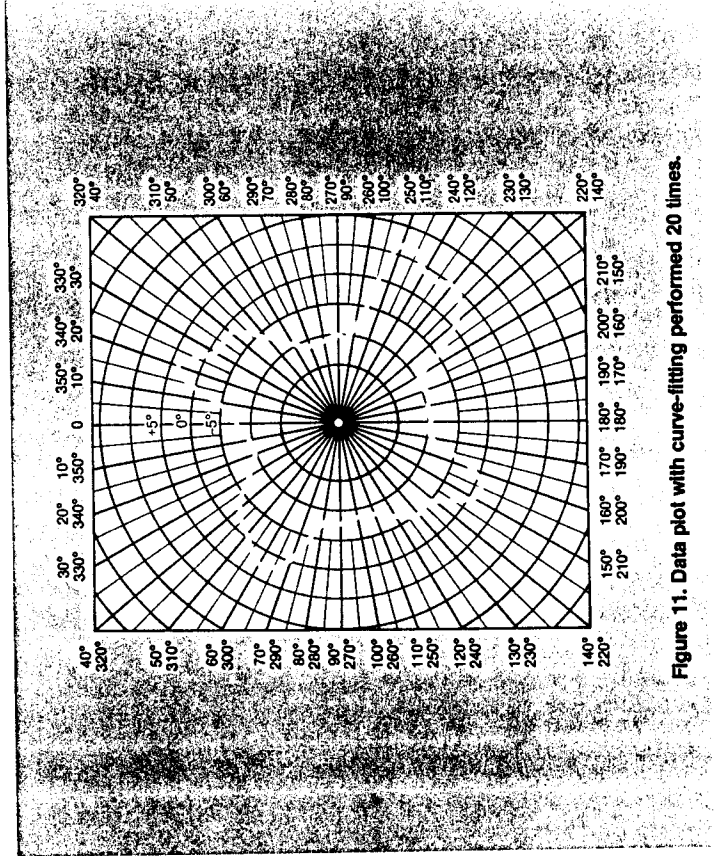


Figure 11. Data plot with curve-fitting performed 20 times

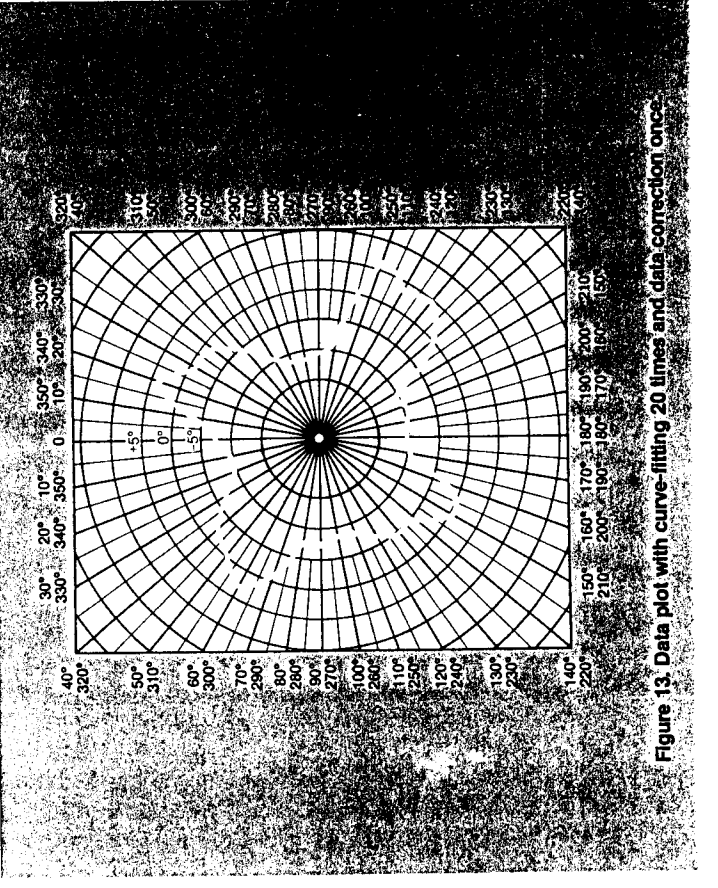


Figure 13. Data plot with curve-fitting 20 times and data correction once

The data values in Figure 10 represent a closer approximation to the corrections needed for the DF, but are they accurate enough? The data could be run back through the curve fitting algorithm to determine if there is much change caused by the second processing. By rerunning the data through the curve-fitting algorithm, 7 points are again being combined to give an approximation; these 7 points were already a combination of 7 points, which means that now a total of 13 points are used to make the approximation. This should give a smooth curve and a more accurate data value. If this data is run through the smoothing algorithm a total of 59 times, then all 360 data points will have contributed to each data value.

The optimum number of times to run the data through the curve-smoothing algorithm depends on the cohesiveness of the data. If the data follows a small symmetrical sine wave, the smoothing will be done rapidly. If the data is disorderly, jumping around without much pattern, then it may take many passes to smooth the data. It may even be desirable to not correct the data to the original data values. As a general rule of thumb, for a coherent data set and moderate accuracy, the smoothing function should be run between 2 and 5 times.

Figure 11 is a plot of the same base data as Figures 8, 9, and 10. In Figure 11 the data has been passed through the smoothing algorithm and correcting algorithm 20 times.

In Figure 10, notice the peaking of the error lobe at 20 degrees. In Figure 11, the same lobe is not as peaked, it is a gentler curve as would be expected of a

continuous function. Also, compare the data value at 15 degrees. The value has changed from .3 to 1. This is a change of .7 degrees, an equivalent error of over 617 feet at a range of 10 miles. The number of times the data needed to go through the smoothing function to get a fair degree of roundness is an indication that the data was somewhat erratic to start with. The curve-fitted values of Figure 11 are probably as accurate as can be expected for the DF correction values.

If the correction algorithm is not used, then some other form of compensation should be used, such as multiplying the smoothed data by a constant that is based on the number of times the data has been smoothed. If no compensation is used, all the lobes will slowly disappear. Figure 12 shows the amount of "desensitizing" of the data for doing the curve fitting 20 times. The comparison of this plot with Figure 11 is a good demonstration of the need for compensation of some sort.

Another problem similar to forgetting to do the correction algorithm at all, is not correcting the data often enough. Figure 13 shows this condition on the same data set. The data for this plot was smoothed 20 times, and then corrected to the original data only once. Notice the choppiness of the plot. The pattern no longer appears smooth and predictable as it should be. A general rule for data correcting or compensating is to compensate each time a data set is smoothed.

Figure 14 shows a flow diagram of a full data reduction program. This algorithm allows for operator intervention and extra smoothing required for the erratic data set.

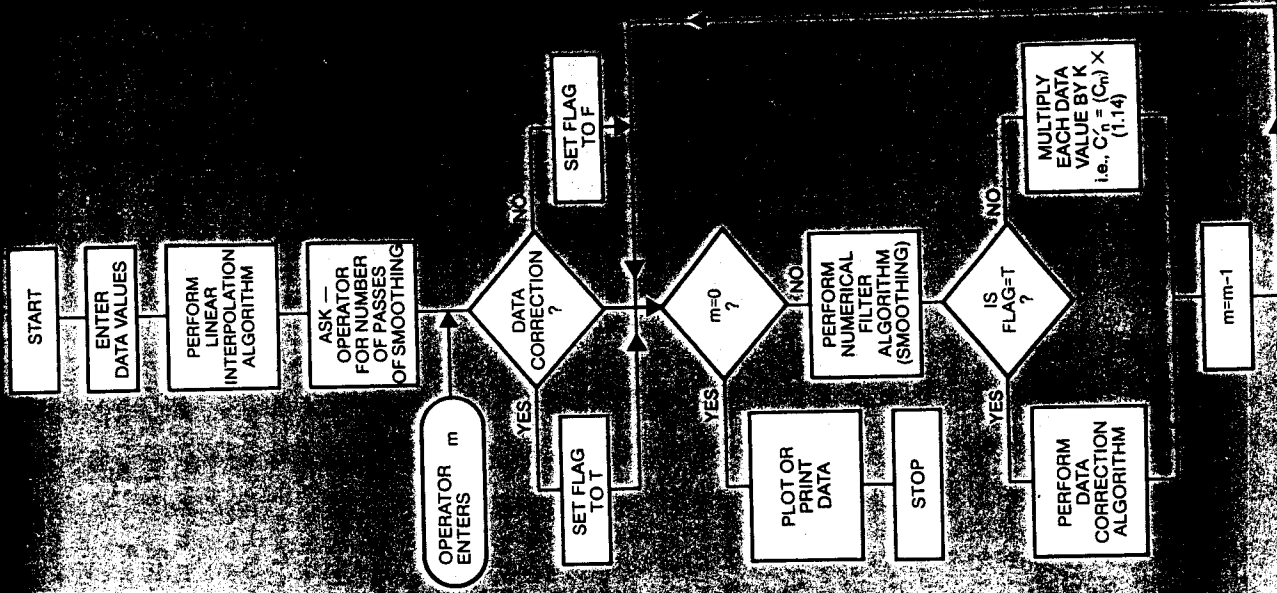


Figure 14. Flowchart for data reduction program.

Conclusion

The process of creating a correction table (or testing a DF system) is summarized below:

1. Determine the location to run the test.
2. Set up the test area, including sketches and comments about the area.
3. Measure the LOB and determine the correction (using true LOB - DF Reading).
4. Plot the data and calculate the SD (use visual correlation as final acceptance).
5. Perform the χ^2 test if the SD is larger than the specification (or the historical value) to determine if the data set is acceptable (when testing overall accuracy, use the data of 5 or more frequencies).

With the five steps listed above, the data required to test a DF site or to construct a correction table will be obtained. The following steps are necessary only when constructing a correction table:

1. Average together any data set of identical frequency to reduce random errors.
2. Determine the amount of accuracy needed.
 - a. If linear interpolation is good enough, then just create a table with the original data, and go to step 3.
 - b. If more resolution is required, perform the curve smoothing and correction algorithms until desired smoothness is achieved (between 2 and 5 times for cohesive data).
3. Create the correction table.

Insufficient detail paid to any of the first three steps could have a detrimental effect on the whole process. Care and accuracy should be exercised in all steps.

Glossary of Terms

1. **Correction Table** — A convenient and systematic display of correction values.
2. **Curve fitting** — The process of manipulating the linear interpolated data to form a smooth curve.
3. **Data values** — The value taken during DF tests to be used as correction values; the true LOB minus the DF reading.
4. **Interpolation** — The process of calculating approximate values between two known values.
5. **LOB (Line of Bearing)** — The bearing displayed or calculated from the display of a direction finder.
6. **Numerical filtering** — See curve fitting.
7. **RMS (Root Mean Square)** — A value indicating the average amount of deviation in the data values.
8. **SD (Standard Deviation)** — A statistical measure of the deviation of data values around the average of the data.
9. **Wavelength** — The distance from one point on a wave to the same point on the preceding wave. To calculate the wavelength, the distance in feet is derived by dividing 984 by the frequency in megahertz.
10. χ^2 — "Chi squared," a statistical test of acceptance based on the value of the standard deviation.

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Author:



James B. Harrington

Mr. Harrington is a Member of the Technical Staff, Receiver Department of the CEI Division.

He has worked on several direction finders, including a ruggedized man-pack WJ-8975, and rack-mount WJ-8971A processors. He is presently involved in designing a microprocessor controlled direction finder, which is the next generation of direction finders for Watkins-Johnson Company.

Previously, Mr. Harrington was a co-op student at Georgia Institute of Technology. His work experience there includes: microprocessor development and work with several radars at the Experimental Station, Assistant Test Engineer with the propulsion test stands at NASA in Huntsville, Alabama, and Assistant Engineer at the Integrated Circuit Development Lab, also with NASA in Huntsville, Alabama.

Mr. Harrington holds a Bachelor of Electrical Engineering degree from Georgia Institute of Technology.